

Half-vortices in polariton condensates

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It is shown that vortices in linearly polarized polariton condensates in planar semiconductor microcavities carry two winding numbers (k, m) . These numbers can be either integer or half-integer simultaneously. Four half-integer vortices $(1/2, 1/2)$, $(-1/2, -1/2)$, $(1/2, -1/2)$, and $(-1/2, 1/2)$ are anisotropic, possess the smallest energy, and define the Kosterlitz-Thouless transition temperature. The condensate concentration remains finite within the core of half-vortex and the condensate polarization becomes fully circular in the core center.

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Introduction.—Recent observations of exciton-polariton condensation in semiconductor microcavities [1, 2] revealed the formation of condensates with a well-defined linear polarization. The polarization is build-up as a result of minimization of the energy of polariton-polariton repulsion H_{int} [3, 4],

$$H_{\text{int}} = \frac{1}{2} \int d^2r \{ (U_0 - U_1)(\psi^* \cdot \psi)^2 + U_1 |\psi^* \times \psi|^2 \}. \quad (1)$$

Here the integration is over the microcavity plane within the excitation spot and the polariton condensate wave function (the order parameter) is written as a complex two-dimensional vector $\psi(\mathbf{r})$. This vector describes the in-plane component of electric field of the polariton condensate and it is normalized to the condensate concentration $n = (\psi^* \cdot \psi)$. The polariton repulsion is characterized by two interaction constants, U_0 and U_1 [4]. They are typically related as $U_0 > U_1 > 0$, so that at a fixed polariton concentration the minimum of H_{int} is reached at $\psi^* \times \psi = 0$, i.e., at a linear polarization.

While the condensation has been observed in [1, 2] for the case of localized polaritons, it is of key importance to understand the structure and polarization properties of topological defects (vortices) in uniform polariton condensates. First, it is vortex-antivortex unbinding that defines the critical temperature T_c for the condensation transition in two-dimensions (2D) according to the Kosterlitz-Thouless scenario [5]. And the estimations of T_c given till now [6] did not take into account the polarization degree of freedom of polaritons. Secondly, since vortices are topologically stable objects, they can be used as long-living optical memory elements. The vortices studied below can be applied for polarization sensitive optical computing.

Winding numbers and vortex interactions.—I begin with the classification of vortices in polariton condensates. Following the general approach [7], it is necessary to distinguish topologically different mappings of a closed path in the order parameter space into a closed path around the vortex core in the microcavity plane. Since a linear polarization vector can be written in general as

$$\psi_{\text{lin}} = \{\psi_x, \psi_y\} = \sqrt{n} e^{i\theta} \{\cos \eta, \sin \eta\}, \quad (2)$$

the order parameter is defined by two angles, $\eta(\mathbf{r})$ and $\theta(\mathbf{r})$. The topology of the order parameter space, however, does not coincide with the topology of a torus, because the points η, θ and $\eta + \pi, \theta + \pi$ should be identified. It is more convenient to imagine the order parameter space as an infinite chessboard shown in Fig. 1. A vortex is defined by a line connecting two equivalent points on this chessboard, and it is characterized by two winding numbers, k and m , such that the angles are changed as $\eta \rightarrow \eta + 2\pi k$, $\theta \rightarrow \theta + 2\pi m$ by encircling the vortex core around. It is seen that the winding numbers k and m can take either integer or half-integer values together. Basic vortices carry half-integer values $k, m = \pm 1/2$. Corresponding changes of the order parameter are shown by four thick arrows in Fig. 1. Other vortices are reduced to superpositions of four basic half-vortices.

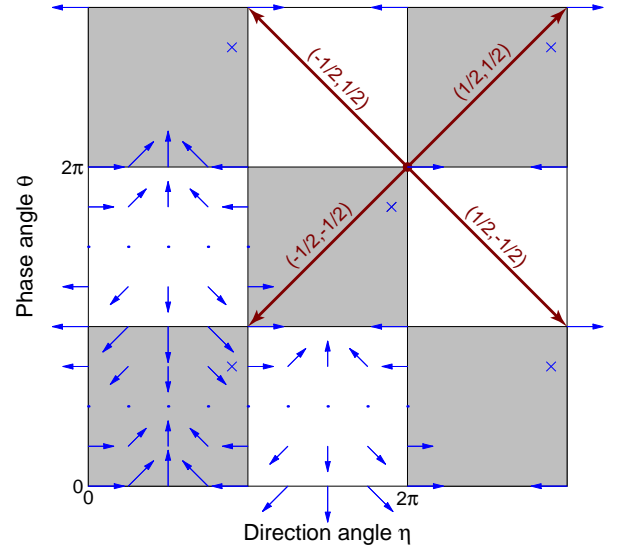


FIG. 1: Illustrating the order parameter space of polariton condensate. Blue arrows show $\text{Re}\{\psi\}$. Similar points in the black or white squares (as, e.g., the points marked with small crosses) represent identical values of the order parameter. Thick red arrows show four topologically different changes of the order parameter when one encircles the cores of four basic half-vortices.

Existence of half-integer vortices is a general feature of multi-component condensates [8], but usually they carry only one half-integer topological charge originating from the phase angle change. E.g., the half-vortices in $^3\text{He-A}$ generate rotations $\mathbf{d} \rightarrow -\mathbf{d}$ of the spin quantization axis \mathbf{d} , accompanied by the phase change $\theta \rightarrow \theta \pm \pi$ [9]. Similar half-vortices have been discussed for the three-component spinor $s = 1$ atomic condensates [10]. For a 3D vector \mathbf{d} , different trajectories connecting two antipodes on the sphere, \mathbf{d} and $-\mathbf{d}$, can be continuously transformed one to another, so that all half-vortices with the phase changing by π are topologically equivalent. Contrary, the polariton condensates are two-component, and the clockwise and counterclockwise rotations of the 2D real vector $\{\cos\eta, \sin\eta\}$ are topologically distinct. This gives rise to the second topological charge.

Half-vortices with two winding numbers have been predicted for $^3\text{He-A}$ in the parallel-plate geometry [11] where the spin quantization axis \mathbf{d} is restricted to be two-dimensional. In this case, however, the sign of phase change is fixed and only two half-vortices can be present in the same configuration, while all four half-vortices can coexist in the polariton condensate.

The total energy of polariton condensate is

$$H = \int d^2r \left[-\frac{\hbar^2}{2m^*} (\psi^* \cdot \Delta \psi) - \mu (\psi^* \cdot \psi) \right] + H_{\text{int}}, \quad (3)$$

where m^* is the polariton effective mass at the bottom of the lower polariton branch and $\mu = (U_0 - U_1)n$ [4] is the chemical potential. The vortex energy consists of the core energy and the elastic energy E_{el} . The latter comes from polariton kinetic energy far away from the core, where Eq. (2) can be used. This gives

$$E_{\text{el}} = \frac{1}{2} \rho_s \int d^2r [(\nabla\eta)^2 + (\nabla\theta)^2] \quad (4)$$

with rigidity $\rho_s = \hbar^2 n / m^*$. The vortex elastic energies and vortex interactions in logarithmic approximation can be calculated from (4) similarly to the usual case [12].

The energy of a single vortex is $E_{\text{el}}^{(s)} = \pi \rho_s (k^2 + m^2) \ln(R/a)$, where $a = \hbar / (2m^* \mu)^{1/2}$ is the core radius (see below) and $R \gg a$ is the radius of the polariton excitation spot. Clearly, the elastic energy of the half-vortex is twice smaller than the energy of a usual vortex [e.g., of the $(0, 1)$ vortex]. The energy $E_{\text{el}}^{(p)}$ of the pair of vortices (k_1, m_1) and (k_2, m_2) separated by distance r , such that $a \ll r \ll R$, is

$$E_{\text{el}}^{(p)} = \pi \rho_s [(k_1 + k_2)^2 + (m_1 + m_2)^2] \ln(R/a) + 2\pi \rho_s (k_1 k_2 + m_1 m_2) \ln(a/r). \quad (5)$$

It is seen that only half-vortices with the same sign of the product km interact. This sign, as we see below, defines the sign of circular polarization in the center of the vortex core—it is right-circular for $\text{sgn}(km) = +1$

and left-circular for $\text{sgn}(km) = -1$. In particular, there is attraction between the right half-vortex and the right anti-half-vortex, $(1/2, 1/2)$ and $(-1/2, -1/2)$, as well as between the left ones, $(1/2, -1/2)$ and $(-1/2, 1/2)$. On the same time, the right half-vortices do not interact with the left half-vortices, and, for example, the energy of the pair $(1/2, 1/2)$ and $(1/2, -1/2)$ simply equals to the energy of a single $(1, 0)$ vortex.[13]

At a finite temperature equal numbers of left and right half-vortex pairs are excited. Since the subsystems of left and right vortices do not interact, they evolve independently with increasing temperature and are subject to the Kosterlitz-Thouless transition at $T_c = (\pi/4)\rho_s$. This temperature is twice smaller than the critical temperature for spin-less bosons due to the double reduction of the single half-vortex energy. Clearly, this estimation of T_c does not take into account the depletion of the condensate, but it shows the necessity to allow for the polarization degree of freedom for any realistic calculations of the transition temperature. The critical temperature is expected to be modified also by non-parabolicity of polariton kinetic energy, as well as by the account for the longitudinal-transverse splitting of polariton branch that lead to the coupling between left and right vortices. Note also that the symmetry between left and right vortex subsystems is broken in applied magnetic field.

Polarization texture of half-vortex core.—The chemical potential μ is found experimentally as a blue-shift of polariton luminescence line due to the polariton condensation, and, typically, $\mu \lesssim 1 \text{ meV}$ [1, 14]. Due to a very small value of the polariton effective mass m^* , ranging from 10^{-4} to 10^{-5} of the free electron mass [15], the size of half-vortex core is quite big, $a \gtrsim 1 \mu\text{m}$. Therefore, the polarization texture within the core region of a half-vortex can be observed by means of near-field luminescence spectroscopy. This texture is studied in this subsection.

In what follows, only the basic half-vortices with $|k| = |m| = 1/2$ will be considered. In this case, the order parameter can be written in cylindrical coordinates as

$$\psi_{\text{hv}} = \sqrt{n} [\mathbf{A}(\phi) f(r/a) - i \mathbf{B}(\phi) g(r/a)], \quad (6)$$

where the azimuthal dependencies are given by

$$\mathbf{A}(\phi) = e^{im\phi} \{\cos(k\phi), \sin(k\phi)\}, \quad (7a)$$

$$\mathbf{B}(\phi) = \text{sgn}(km) e^{im\phi} \{\sin(k\phi), -\cos(k\phi)\}, \quad (7b)$$

and the radial functions $f(r/a)$ and $g(r/a)$ are found from minimization of the vortex energy, i.e., from the equation $\delta H / \delta \psi = 0$. Using the relations

$$\mathbf{A}''(\phi) = i \mathbf{B}''(\phi) = -\frac{1}{2} (\mathbf{A} + i \mathbf{B}), \quad (8a)$$

$$\mathbf{A} - i \mathbf{B} = \{1, \text{sgn}(km) i\}, \quad (8b)$$

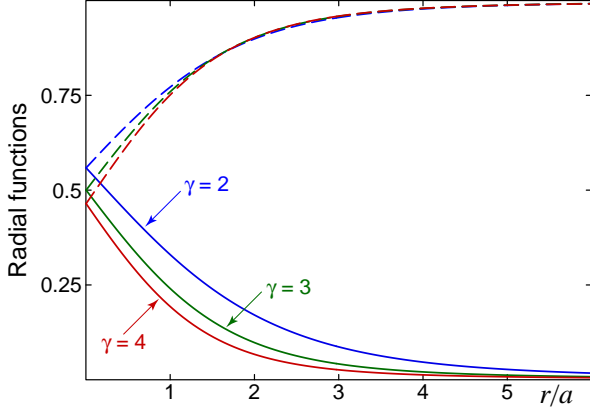


FIG. 2: Showing the half-vortex radial functions $f(r/a)$ (dashed) and $g(r/a)$ (solid) for three values of the interaction parameter γ .

one obtains ($\xi = r/a$)

$$f'' + \frac{1}{\xi}f' - \frac{1}{2\xi^2}(f - g) + [1 - f^2 - \gamma g^2]f = 0, \quad (9a)$$

$$g'' + \frac{1}{\xi}g' - \frac{1}{2\xi^2}(g - f) + [1 - g^2 - \gamma f^2]g = 0, \quad (9b)$$

where $\gamma = (U_0 + U_1)/(U_0 - U_1)$.

Equations (9a,b) are symmetric with respect to interchange of radial functions f and g . The half-vortex, however, is described by an asymmetric solution. Since the polarization should become linear at large distances, one has $f(\infty) = 1$ and $g(\infty) = 0$. This way at $r \gg a$ the solution (6) transforms into (2) with $\eta = k\phi$ and $\theta = m\phi$. On the other hand, in the half-vortex center one has to demand $f(0) = g(0)$ in order to remove the divergences produced by terms $(f - g)/2\xi^2$ in Eqs. (9). Taking into account the relation (8b) it is clear that the polarization is fully circular in the core center ($r = 0$), and the sign of circular polarization is given by $\text{sgn}(km)$, as was mentioned above.

Equations (9) have simple solutions in the particular case $U_1 = U_0/2$, i.e., for $\gamma = 3$. Namely, $f(\xi) = [1 + h(\xi)]/2$ and $g(\xi) = [1 - h(\xi)]/2$, where $h(\xi)$ is the radial function of a usual vortex in spin-less condensate [16]. Since $h(\xi)$ is monotonously decaying from 1 at $\xi \rightarrow \infty$ to 0 at $\xi = 0$, the polariton concentration $n(r) = f^2(r/a) + g^2(r/2)$ is decreasing from its unperturbed value $n \equiv n(\infty)$ to $n/2$ in the half-vortex center.

In general, for realistic values of γ [17] the behavior of f and g remains close to the above case. The numerical solutions of Eqs. (9) are shown in Fig. 2. Note that while the asymptotic behavior of $f(\xi)$ is γ -independent for large ξ , $f(\xi) \simeq 1 - (4\xi^2)^{-1}$, the second radial function behaves asymptotically as $g(\xi) \simeq [2(\gamma - 1)\xi^2]^{-1}$. Therefore, the effective size of the half-vortex core grows with decreasing γ and diverges at $\gamma \rightarrow 1$, i.e., when $U_1 \rightarrow 0$.

This reflects the fact that polaritons are not expected to exhibit the superfluid transition for $U_1 = 0$. In this case the polariton system has $O(4)$ symmetry and the order is destroyed at any finite temperature according to the nonlinear σ -model analysis.

Concerning the polarization texture of the half-vortices given by Eqs. (6) and (7), it should be noted that different half-vortices can be transformed to each other by applying two symmetry operations, (i) the 2D inversion ($x \rightarrow x, y \rightarrow -y$), and (ii) the time inversion or complex conjugation. Each of these operations used separately transforms the right $(1/2, 1/2)$ half-vortex into one of two left half-vortices, while the subsequent application of both operations yields the $(-1/2, -1/2)$ anti-half-vortex. Clearly, these symmetry operations leave the radial functions $f(r/a)$ and $g(r/a)$ unchanged.

The half-vortices possess two anisotropic polarization textures shown in Fig. 3(a,b). These figures rely on the usual representation of the time-dependent electric field as $\mathbf{E}(t) \propto \text{Re}\{\psi e^{-i\omega t}\}$, where ω is given by the bare frequency ω_0 of the lower-branch polariton in microcavity blue-shifted by the chemical potential μ due to the polariton-polariton repulsion, $\omega = \omega_0 + \mu$. It is seen that when one approaches the core center linear polarizations convert into elliptical ones. The circular polarization degree is given by

$$\rho_{\text{circ}} = \text{sgn}(km) \frac{2fg}{f^2 + g^2}, \quad (10)$$

and $|\rho_{\text{circ}}|$ increases from 0 to 1 with decreasing r . In spite of the dependence of the directions of main axes of each elliptical polarization on the azimuthal angle ϕ , all elliptical polarizations meet in phase at $r = 0$ and transform into the same circular polarization. Note also that each image in Fig. 3 represents two half-vortices, one with the clockwise rotation of polarization vector and the other with the counterclockwise rotation.

The above analysis of half-vortex properties is based on minimization of the polariton energy subject to specific boundary conditions, which is valid for the case of polariton condensate in quasi-thermal equilibrium. Such thermalized polariton condensates have been recently observed [1, 18]. In these conditions the half-vortex pairs can both appear spontaneously and be artificially excited. In particular, the half-vortex and anti-half-vortex pair can be obtained by shining the uniform linearly polarized condensate with two circularly polarized pulses having appropriate intensities, spot radii, and separation. Another option consists in the excitation of a $(0, 1)$ vortex using the optical parametric oscillator setup [19]. This vortex involves only phase change and is topologically equivalent to the pairs of $(1/2, 1/2)$ and $(-1/2, 1/2)$ half-vortices, or to the pair of $(-1/2, 1/2)$ and $(1/2, 1/2)$ ones, so it can decay in one of these pairs.

Finally, it should be noted that similar half-vortices with two topological charges can be present in mixtures of

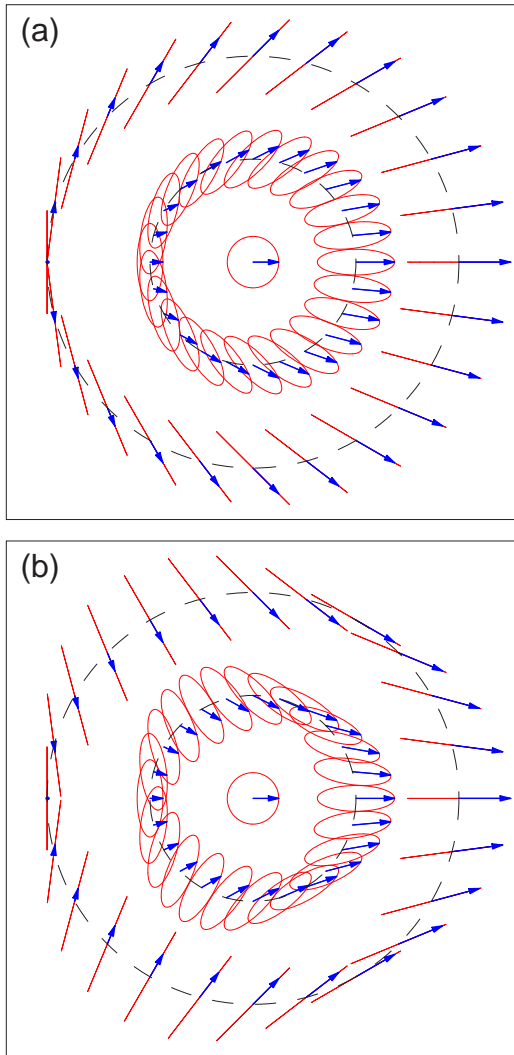


FIG. 3: Showing two distinct core textures of basic half-vortices. The case (a) is realized for the right $(1/2, 1/2)$ and the left $(1/2, -1/2)$ half-vortex, while the case (b)—for right $(-1/2, -1/2)$ and the left $(-1/2, 1/2)$ half-vortex. Arrows indicate the instant polarization at two values of $r = \text{const}$ (dashed circles) and in the core center $r = 0$. The polarization vectors change in time following red solid lines.

two hyperfine spin states of ^{87}Rb atomic condensates [20], provided the order parameter manifold is given by two angles as in Fig. 1. In this case the half-vortices should also control the Kosterlitz-Thouless transition observed in these condensates [21].

Conclusions.—Polarization vortices in linearly polarized polariton condensate have been analyzed. It has been shown that four half-vortices with the direction and phase winding numbers equal to $\pm 1/2$ have the smallest energy. These half-vortices form two subsystems exhibiting independent Kosterlitz-Thouless transitions at the same temperature. The condensate polarization becomes elliptical within the core of a half-vortex and converts to fully circular in the core center.

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